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Abstract

This report is concerned with the derivation of the equations of motion for the Spacecraft Control Laboratory Experiment (SCOLE). For future reference, the equations of motion of a similar structure orbiting the earth are also derived. The structure is assumed to undergo large rigid-body maneuvers and small elastic deformations. A perturbation approach is proposed whereby the quantities defining the rigid-body maneuver are assumed to be relatively large, with the elastic deformations and deviations from the rigid-body maneuver being relatively small. The perturbation equations have the form of linear equations with time-dependent coefficients. An active control technique can then be formulated to permit maneuvering of the spacecraft and simultaneously suppressing the elastic vibration.

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1. Introduction

We shall first derive Lagrange's equations of motion for the spacecraft of Fig. 1 regarding the structure as orbiting about the earth and then modify these equations so as to describe the laboratory experiment. In the derivation, the shuttle is treated as a rigid body and the beam and antenna as flexible, distributed parameter systems. The equations of motion can be further modified for the case of a rigid antenna.

The equations describing a maneuver of a rigid space structure consist of nonlinear ordinary differential equations. On the other hand, the equations describing the small elastic displacements of a flexible structure, relative to the rigid-body maneuver are linear partial differential equations. Hence, the complete equations of motion describing a flexible body during a maneuver represent a set of nonlinear hybrid differential equations.

Hybrid systems possess an infinite number of degrees of freedom. In practice, however, it is necessary to reduce the number of degrees of freedom to a finite one, which implies spatial discretization and truncation. Substructure synthesis often proves useful as a method of discretization and truncation, particularly in the case of distributed substructures. Even in the case of discrete substructures, a set of linearly independent vectors can be used as admissible vectors to reduce the number of equations of motion.

In this report, we propose a perturbation technique whereby the flexible spacecraft maneuver is assumed to consist of a combination of a rigid-body maneuver and small motions including rigid-body deviations from the rigid-body maneuver and elastic vibrations. Regarding the

rigid-body maneuver as known, the perturbation equations for the vibration control reduce to a set of linear ordinary differential equations with known time-varying coefficients.

2. Equations of Motion of the Spacecraft

It is convenient to refer the motion of the spacecraft to a given reference frame $x_0y_0z_0$, where the frame can be regarded as being embedded in the rigid shuttle. The reference frame has six degrees of freedom, three rigid-body translations and three rigid-body rotations.

We propose to derive the equations of motion by means of the Lagrangian approach. To this end, we must first obtain expressions for the kinetic energy, the potential energy and the virtual work. Considering Fig. 1 and denoting the position of the origin 0 of the frame $x_0y_0z_0$ by the vector $\underset{\sim}{R}$ and the position of a point S in the shuttle relative to 0 by $\underset{\sim}{r}$, the position of S relative to the inertial frame XYZ is $\underset{\sim}{R}_S = \underset{\sim}{R} + \underset{\sim}{r}$. Moreover, denoting by $\underset{\sim}{a}$ the vector from 0 to a nominal point A on the appendage and by $\underset{\sim}{u}$ the elastic displacement vector of the point, the position of A in the displaced configuration is $\underset{\sim}{R}_A = \underset{\sim}{R} + \underset{\sim}{a} + \underset{\sim}{u}$. It must be noted that the vectors $\underset{\sim}{r}$, $\underset{\sim}{a}$ and $\underset{\sim}{u}$ are likely to be measured relative to axes $x_0y_0z_0$. In view of the above, the velocity of a point S in the shuttle is

$$\dot{\underset{\sim}{R}}_S = \dot{\underset{\sim}{R}} + \underset{\sim}{\omega} \times \underset{\sim}{r} \qquad (1)$$

where $\dot{\underset{\sim}{R}}$ is the translational velocity and $\underset{\sim}{\omega}$ is the angular velocity of the frame $x_0y_0z_0$ with respect to the inertial frame. Similarly, the velocity of a point A in the appendage is

$$\dot{\underset{\sim}{R}}_A = \dot{\underset{\sim}{R}} + \underset{\sim}{\omega} \times (\underset{\sim}{a} + \underset{\sim}{u}) + \dot{\underset{\sim}{u}} \qquad (2)$$

where $\dot{\underset{\sim}{u}}$ is the elastic velocity of the point relative to the $x_0y_0z_0$ frame. Hence, the kinetic energy of the spacecraft is

$$T = \frac{1}{2}\int_{m_S} |\dot{\underset{\sim}{R}}_S|^2 dm_S + \frac{1}{2}\int_{m_A} |\dot{\underset{\sim}{R}}_A|^2 dm_A$$

$$= \frac{1}{2}\int_{m_S} |\dot{\underset{\sim}{R}} + \underset{\sim}{\omega} \times \underset{\sim}{r}|^2 dm_S + \frac{1}{2}\int_{m_A} |\dot{\underset{\sim}{R}} + \underset{\sim}{\omega} \times (\underset{\sim}{a} + \underset{\sim}{u}) + \dot{\underset{\sim}{u}}|^2 dm_A$$

$$= \frac{1}{2} m|\dot{\underset{\sim}{R}}|^2 + \frac{1}{2} \underset{\sim}{\omega}^T I_0 \underset{\sim}{\omega} + \dot{\underset{\sim}{R}}\cdot(\underset{\sim}{\omega} \times \underset{\sim}{S}_0) + \frac{1}{2}\int_{m_A} |\dot{\underset{\sim}{u}}|^2 dm_A$$

$$+ \dot{\underset{\sim}{R}}\cdot[\int_{m_A} \dot{\underset{\sim}{u}}\, dm_A + \underset{\sim}{\omega} \times \int_{m_A} \underset{\sim}{u}\, dm_A] + \int_{m_A} \dot{\underset{\sim}{u}}\cdot(\underset{\sim}{\omega} \times \underset{\sim}{a}) dm_A$$

$$+ \int_{m_A} (\underset{\sim}{\omega} \times \underset{\sim}{a})\cdot(\underset{\sim}{\omega} \times \underset{\sim}{u}) dm_A + \frac{1}{2}\int_{m_A} |\underset{\sim}{\omega} \times \underset{\sim}{u}|^2 dm_A$$

$$+ \int_{m_A} \dot{\underset{\sim}{u}}\cdot(\underset{\sim}{\omega} \times \underset{\sim}{u}) dm_A \qquad (3)$$

where

$$\underset{\sim}{S}_0 = \int_{m_S} \underset{\sim}{r}\, dm_S + \int_{m_A} \underset{\sim}{a}\, dm_A \qquad (4)$$

and m_S, m_A and m are the masses of the shuttle, appendage and entire spacecraft, respectively. Also, I_0 is the total mass moment of inertia matrix of the undeformed structure about point 0. Note that $|\underset{\sim}{x}|^2$ denotes the inner product $\underset{\sim}{x}\cdot\underset{\sim}{x}$.

The potential energy is due to the combined effects of gravity and strain energy. Assuming that the origin of the inertial coordinate system coincides with the center of the gravitational field, the gravitational potential can be expressed as

$$V_g = - Gm_e[\int_{m_S} |\underset{\sim}{R} + \underset{\sim}{r}|^{-1} dm_S + \int_{m_A} |\underset{\sim}{R} + \underset{\sim}{a} + \underset{\sim}{u}|^{-1} dm_A] \qquad (5)$$

where m_e is the mass of the earth and G is the gravitational constant.

The strain energy can be expressed as an energy inner product symbolized by [,] (Ref. 1). The total potential energy then becomes

$$V = \frac{1}{2}[\underset{\sim}{u},\underset{\sim}{u}] + V_g \tag{6}$$

The virtual work is due to external forces, including control forces. Denoting by $\underset{\sim}{f}_S$ the force vector per unit volume of the shuttle and by $\underset{\sim}{f}_A$ the force vector per unit volume of the appendage, we can write the virtual work as

$$\delta W = \int_{D_S} \underset{\sim}{f}_S \cdot \delta \underset{\sim}{R}_S dD_S + \int_{D_A} \underset{\sim}{f}_A \cdot \delta \underset{\sim}{R}_A dD_A \tag{7}$$

where D_S and D_A are the domains of the shuttle and appendage, respectively.

Before deriving the equations of motion, we consider certain simplifying assumptions. To this end, we estimate the maximum possible angular velocities by ignoring the effects of the appendage and examining the prescribed maneuvers of the shuttle alone. For the 20^o maneuver about the x_0 axis, applying maximum torque, the maximum angular velocity is approximately .06 rad/s. For the 90^o maneuver about the z_0 axis the maximum angular velocity is .047 rad/s. If the elastic displacements are small, then the last two terms of Eq. (3) are of higher order and can be neglected. The third to last term of Eq. (3) will be retained, despite leading to nonlinear terms, because the magnitude of the factor multiplying the independent variables $\underset{\sim}{u}$ and $\underset{\sim}{\omega}$ tends to offset the smallness of the independent variables. Next, we express the elastic displacements in the form of linear combinations of admissible functions, or

$$\underset{\sim}{u} = \Phi \underset{\sim}{q} \tag{8}$$

where Φ is a matrix of space-dependent admissible functions and $\underset{\sim}{q}$ is a vector of time-dependent generalized coordinates. Introducing Eq. (8) into Eq. (3) and neglecting the last two terms in Eq. (3), the kinetic energy takes the matrix form

$$\begin{aligned} T \cong\ & \frac{1}{2} m\dot{\underset{\sim}{R}}^T\dot{\underset{\sim}{R}} + \frac{1}{2} \underset{\sim}{\omega}^T I_0 \underset{\sim}{\omega} + \dot{\underset{\sim}{R}}^T C^T \tilde{S}_0 \underset{\sim}{\omega} + \frac{1}{2} \dot{\underset{\sim}{q}}^T M_A \dot{\underset{\sim}{q}} \\ & + \dot{\underset{\sim}{R}}^T C^T \bar{\Phi} \dot{\underset{\sim}{q}} + \dot{\underset{\sim}{R}}^T C^T \tilde{\omega}^T \bar{\Phi} \underset{\sim}{q} + \dot{\underset{\sim}{q}}^T \tilde{\Phi}^T \underset{\sim}{\omega} + \underset{\sim}{\omega}^T \int_{m_A} \tilde{a}^T \tilde{\omega}^T \Phi \, dm_A \underset{\sim}{q} \end{aligned} \tag{9}$$

where

$$\bar{\Phi} = \int_{m_A} \Phi \, dm_A, \quad \tilde{\Phi}^T = \int_{m_A} \Phi^T \tilde{a} \, dm_A \tag{10a,b}$$

and

$$M_A = \int_{m_A} \Phi^T \Phi \, dm_A \tag{10c}$$

is the mass matrix of the appendage. The symbol C represents a rotation matrix from the inertial frame to the $x_0y_0z_0$ frame and its elements are nonlinear functions of a set of Euler angles $\underset{\sim}{\alpha}$. The tilde over a given vector such as $\underset{\sim}{v}$ denotes a skew symmetric matrix of the form

$$\tilde{v} = \begin{bmatrix} 0 & v_z & -v_y \\ -v_z & 0 & v_x \\ v_y & -v_x & 0 \end{bmatrix} \tag{11}$$

Recognizing that the magnitude of $\underset{\sim}{R}$ is large and $\underset{\sim}{u}$ is small in comparison with the other vectors in Eq. (5) and ignoring terms of order higher than three, a binomial expansion permits us to write

$$V_g \cong - Gm_e[m|\underset{\sim}{R}|^{-1} - \underset{\sim}{R}\cdot(\underset{\sim}{S}_0 + \int_{m_A} \underset{\sim}{u} \, dm_A)|\underset{\sim}{R}|^{-3}] \tag{12}$$

Introducing Eq. (8) into Eq. (6) and considering Eq. (12), the potential energy can be written in the matrix form

$$V = \frac{1}{2}\, g^T K_A g - \frac{Gm_e m}{|R|} + \frac{Gm_e}{|R|^3}\, R^T C^T (S_0 + \bar{\Phi} g) \tag{13}$$

where

$$K_A = [\phi, \phi] \tag{14}$$

is the stiffness matrix of the appendage. The virtual work can be shown to have the expression

$$\delta W = F^T C \delta R + M^T \delta\alpha + Q^T \delta g \tag{15}$$

where

$$\begin{aligned} F &= \int_{D_S} f_S \, dD_S + \int_{D_A} f_A \, dD_A \\ M &= \int_{D_S} \tilde{r}^T f_S \, dD_S + \int_{D_A} \tilde{a}^T f_A \, dD_A + \int_{D_A} \tilde{f}_A \phi \, dD_A g \\ Q &= \int_{D_A} \phi^T f_A \, dD_A \end{aligned} \tag{16}$$

are generalized force vectors in terms of components about x_0, y_0 and z_0.

Without loss of generality, we let point 0 correspond to the center of mass of the spacecraft in its undeformed state, so that the vector S_0 is zero. Then, Lagrange's equations of motion can be written in the symbolic form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{R}}\right) + \frac{\partial V}{\partial R} = C^T F \tag{17a}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\alpha}}\right) - \frac{\partial T}{\partial \alpha} + \frac{\partial V}{\partial \alpha} = M \tag{17b}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{g}}\right) - \frac{\partial T}{\partial g} + \frac{\partial V}{\partial g} = Q \tag{17c}$$

so that, considering Eqs. (9), (13) and (15), the equations of motion for the spacecraft in orbit are

$$m\ddot{\mathbf{R}} + C^T\bar{\Phi}\ddot{\mathbf{q}} + Gm_e\left[\frac{m\mathbf{R}}{|\mathbf{R}|^3} + \frac{C^T\bar{\Phi}\mathbf{q}}{|\mathbf{R}|^3} - \frac{\mathbf{R}(3\mathbf{R}^TC^T\bar{\Phi}\mathbf{q})}{|\mathbf{R}|^5}\right] = C^T\mathbf{F} \tag{18a}$$

$$I_0\dot{\boldsymbol{\omega}} + \tilde{\omega}^T I_0\boldsymbol{\omega} + [\widetilde{C\ddot{\mathbf{R}}}]\bar{\Phi}\mathbf{q} + \tilde{\Phi}\ddot{\mathbf{q}} + \tilde{\omega}^T\tilde{\Phi}\dot{\mathbf{q}} + J(\dot{\boldsymbol{\omega}})\mathbf{q} + J(\omega)\dot{\mathbf{q}}$$

$$+ \tilde{\omega}^T J(\boldsymbol{\omega})\mathbf{q} + \frac{Gm_e}{|\mathbf{R}|^3}[\widetilde{C\mathbf{R}}]\bar{\Phi}\mathbf{q} = \mathbf{M} \tag{18b}$$

$$M_A\ddot{\mathbf{q}} + \bar{\Phi}^T C\ddot{\mathbf{R}} + \tilde{\Phi}^T\dot{\boldsymbol{\omega}} + K_A\mathbf{q} + \int_{m_A}\Phi^T\tilde{\omega}^T\tilde{a}\,dm_A\boldsymbol{\omega} + \frac{Gm_e}{|\mathbf{R}|^3}\bar{\Phi}^T C\mathbf{R} = \mathbf{Q} \tag{18c}$$

where

$$J(\boldsymbol{\omega}) = \int_{m_A}(\tilde{a}\tilde{\omega} + [\widetilde{\tilde{a}\boldsymbol{\omega}}])\Phi\,dm_A \tag{18d}$$

Higher-order terms have been neglected in Eq. (18b), consistent with the preceding discussion concerning the magnitude of the maneuver angular velocities. However, as before, nonlinear terms with large coefficients, such as I_0 and $\mathbf{a}m_A$, have been retained. The position vector $\mathbf{R}$, its time derivatives, and the Euler angles vector $\boldsymbol{\alpha}$ have been considered to be of arbitrary magnitude, with the result that many other nonlinear terms have been retained in Eqs. (18).

3. Equations of Motion for the Laboratory Experiment

In the laboratory experiment, the spacecraft is not actually free in space, but suspended from the ceiling by means of a cable or a beam. The following analysis applies to either case. The support is likely to affect the dynamic characteristics of the system. Hence, in the sequel, the support is added to the free model in the form of an elastic member.

Considering Fig. 2, the position vector for an arbitrary point C on the cable is $\underset{\sim}{R}_C = \underset{\sim}{c} + \underset{\sim}{w}$, where $\underset{\sim}{c}$ is a position vector and $\underset{\sim}{w}$ is the elastic displacement of the cable, both of which are measured with respect to the inertial frame. The position vector for the point 0 is

$$\underset{\sim}{R} = \underset{\sim}{c}_B + \underset{\sim}{w}_B + \underset{\sim}{e} \tag{19}$$

where the subscript B denotes evaluation at the point B and $\underset{\sim}{e}$ is the vector from point B (ball joint) to the point 0 fixed on the "shuttle", measured with respect to the $x_0y_0z_0$ frame. The velocity vector of an arbitrary point C on the cable is then

$$\dot{\underset{\sim}{R}}_C = \dot{\underset{\sim}{w}} \tag{20}$$

and the velocity of point 0 is

$$\dot{\underset{\sim}{R}} = \dot{\underset{\sim}{w}}_B + \underset{\sim}{\omega} \times \underset{\sim}{e} \tag{21}$$

The kinematics for the shuttle body and appendage remain the same as for the unrestrained spacecraft in space. Hence, the kinetic energy for the entire structure is

$$\begin{aligned}
T &= \frac{1}{2}\int_{m_C} |\dot{\underset{\sim}{R}}_C|^2 dm_C + \frac{1}{2}\int_{m_S} |\dot{\underset{\sim}{R}}_S|^2 dm_s + \frac{1}{2}\int_{m_A} |\dot{\underset{\sim}{R}}_A|^2 dm_A \\
&= \frac{1}{2}\int_{m_C} |\dot{\underset{\sim}{w}}|^2 dm_C + \frac{1}{2}\int_{m_S} |\dot{\underset{\sim}{w}}_B + \underset{\sim}{\omega} \times (\underset{\sim}{e} + \underset{\sim}{r})|^2 dm_S \\
&+ \frac{1}{2}\int_{m_A} |\dot{\underset{\sim}{w}}_B + \underset{\sim}{\omega} \times (\underset{\sim}{e} + \underset{\sim}{a} + \underset{\sim}{u}) + \dot{\underset{\sim}{u}}|^2 dm_A \\
&\cong \frac{1}{2}\int_{m_C} |\dot{\underset{\sim}{w}}|^2 dm_C + \frac{1}{2} m|\dot{\underset{\sim}{w}}_B|^2 + \frac{1}{2}\underset{\sim}{\omega}^T I_B \underset{\sim}{\omega} \\
&+ \dot{\underset{\sim}{w}}_B \cdot (\underset{\sim}{\omega} \times \underset{\sim}{S}_B) + (\underset{\sim}{\omega} \times \underset{\sim}{e}) \cdot (\underset{\sim}{\omega} \times \underset{\sim}{S}_B) + \frac{1}{2}\int_{m_A} |\dot{\underset{\sim}{u}}|^2 dm_A \\
&+ \dot{\underset{\sim}{w}}_B \cdot \int_{m_A} \dot{\underset{\sim}{u}}\, dm_A + (\underset{\sim}{\omega} \times \underset{\sim}{e}) \cdot \int_{m_A} \dot{\underset{\sim}{u}}\, dm_A
\end{aligned}$$

$$+ \int_{m_A} \dot{\underset{\sim}{u}} \cdot (\underset{\sim}{\omega} \times \underset{\sim}{a}) dm_A + \int_{m_A} (\underset{\sim}{\omega} \times \underset{\sim}{a}) \cdot (\underset{\sim}{\omega} \times \underset{\sim}{u})\ dm_A \tag{22}$$

where

$$I_B = I_0 + m\tilde{e}^T\tilde{e},\ \underset{\sim}{S}_B = \underset{\sim}{S}_0 + m\underset{\sim}{e} \tag{23}$$

The elastic displacement vectors $\underset{\sim}{u}$ and $\underset{\sim}{w}$ have been assumed to be small. Also, the angular velocity was assumed to be, at most, of the same order of magnitude as in the case of unrestrained spacecraft, so that terms of higher order can once again be neglected.

The expression for the virtual work is given by Eq. (7), but the potential energy must be modified. The acceleration of gravity will be assumed to be constant and can be expressed as $\underset{\sim}{g} = - g\underset{\sim}{u}_Z$ so that the gravitational potential is

$$V_g = \int_{m_C} (\underset{\sim}{c} + \underset{\sim}{w}) \cdot \underset{\sim}{g}\ dm_C + \int_{m_S} (\underset{\sim}{R} + \underset{\sim}{r}) \cdot \underset{\sim}{g}\ dm_S + \int_{m_A} (\underset{\sim}{R} + \underset{\sim}{a} + \underset{\sim}{u}) \cdot \underset{\sim}{g}\ dm_A \tag{24}$$

where $\underset{\sim}{R}$ is defined by Eq. (19) and $\underset{\sim}{u}_Z$ is a unit vector in the Z direction. The elastic potential energy for the system can be expressed as

$$V_E = \frac{1}{2} [\underset{\sim}{u},\underset{\sim}{u}] + \frac{1}{2} [\underset{\sim}{w},\underset{\sim}{w}] \tag{25}$$

where $[\underset{\sim}{w},\underset{\sim}{w}]$ is the energy inner product for the cable, which includes a stiffening effect due to the weight of the "spacecraft". Because a cable has little inherent bending stiffness, this effect can be significant.

As with the appendage in the preceding section, the elastic displacement of the cable can be approximated by a linear combination of admissible functions, or

$$\underset{\sim}{W} = \psi \underset{\sim}{\eta} \tag{26}$$

where ψ is a matrix of space-dependent admissible functions and $\underset{\sim}{\eta}$ is a vector of time dependent generalized coordinates. Introducing Eqs. (8) and (26) into Eq. (22), the kinetic energy takes the matrix form

$$T = \frac{1}{2}\dot{\underset{\sim}{\eta}}^T M_C \dot{\underset{\sim}{\eta}} + \frac{1}{2}\underset{\sim}{\omega}^T I_B \underset{\sim}{\omega} + \dot{\underset{\sim}{\eta}}^T \psi_B^T C^T \tilde{S}_B \underset{\sim}{\omega} + \underset{\sim}{\omega}^T \tilde{e}^T \tilde{S}_0 \underset{\sim}{\omega} + \frac{1}{2}\dot{\underset{\sim}{q}}^T M_A \dot{\underset{\sim}{q}} + \dot{\underset{\sim}{\eta}}^T \psi_B^T C^T \bar{\Phi}\dot{\underset{\sim}{q}}$$

$$+ \underset{\sim}{\omega}^T \tilde{e}^T \bar{\Phi}\dot{\underset{\sim}{q}} + \dot{\underset{\sim}{q}}^T \tilde{\Phi}^T \underset{\sim}{\omega} + \underset{\sim}{\omega}^T \int_{m_A} \tilde{a}^T \tilde{\omega}^T \Phi \, dm_A \, \underset{\sim}{q} \tag{27}$$

where

$$M_C = \int_{m_C} \psi^T \psi \, dm_C + m\psi_B^T \psi_B \tag{27a}$$

is the mass matrix of the cable, including the mass of the structure lumped at the end of the cable. ψ_B denotes the matrix ψ evaluated at B. Introducing Eqs. (8) and (26) into Eqs. (24) and (25) the potential energy can be written in the matrix form

$$V = \underset{\sim}{\eta}^T \bar{\psi}^T \underset{\sim}{g} + \underset{\sim}{S}_B^T C \underset{\sim}{g} + m\underset{\sim}{\eta}^T \psi_B^T \underset{\sim}{g} + \underset{\sim}{q}^T \bar{\Phi}^T C \underset{\sim}{g} + \frac{1}{2}\underset{\sim}{q}^T K_A \underset{\sim}{q} + \frac{1}{2}\underset{\sim}{\eta}^T K_C \underset{\sim}{\eta} \tag{28}$$

where

$$\bar{\psi} = \int_{m_C} \psi \, dm_C \tag{29a}$$

and

$$K_C = [\psi, \psi] \tag{29b}$$

is the stiffness matrix of the support. Considering Eqs. (19) and (25), the virtual work can be expressed as

$$\delta W = \underset{\sim}{F}^T C \psi_B \delta\underset{\sim}{\eta} + \underset{\sim}{M}^T \delta\underset{\sim}{\alpha} + \underset{\sim}{Q}^T \delta\underset{\sim}{q} \tag{30}$$

where all the terms have been defined previously. The effect of the friction of the ball joint can be assumed in the form of an external torque. Hence, we let $\underset{\sim}{M} = \underset{\sim}{M}_c + \underset{\sim}{M}_f$ in Eq. (30), where $\underset{\sim}{M}_c$ is a vector of control moments and $\underset{\sim}{M}_f$ is a vector of frictional moments caused by the ball joint.

Without loss of generality, we assume that point O is the center of mass of the spacecraft, so that $\underset{\sim}{S}_0$ is zero. For the laboratory experiment to be successful the vector $\underset{\sim}{e}$ must be very small, so that the center of mass O tends to coincide with point B. This is difficult to accomplish in practice due to the friction of the ball joint and the difficulties of dynamically balancing a large flexible structure. Realistically, we can expect $\underset{\sim}{e}$ to be small but nonzero. Hence, for simulation purposes, we choose $\underset{\sim}{e}$ to be small, so that the kinetic energy of Eq. (27) can be approximated as follows:

$$T \cong \frac{1}{2}\,\dot{\underset{\sim}{\eta}}^T M_C \dot{\underset{\sim}{\eta}} + \frac{1}{2}\,\underset{\sim}{\omega}^T I_0 \underset{\sim}{\omega} + m\dot{\underset{\sim}{\eta}}^T \psi_B^T C^T \tilde{e} \underset{\sim}{\omega} + \frac{1}{2}\,\dot{\underset{\sim}{q}}^T M_A \dot{\underset{\sim}{q}}$$

$$+ \dot{\underset{\sim}{\eta}}^T \psi_B^T C^T \bar{\Phi} \dot{\underset{\sim}{q}} + \dot{\underset{\sim}{q}}^T \tilde{\Phi}^T \underset{\sim}{\omega} + \underset{\sim}{\omega}^T \int_{m_A} \tilde{a}^T \tilde{\omega}^T \Phi \, dm_A \, \underset{\sim}{q} \tag{31}$$

Lagrange's equations remain in the symbolic form of Eqs. (17), with the exception of Eq. (17a) which must be replaced by

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\underset{\sim}{\eta}}}\right) + \frac{\partial V}{\partial \underset{\sim}{\eta}} = \psi_B^T C^T \underset{\sim}{F} \tag{32}$$

This results in the following equations of motion for the laboratory experiment:

$$M_C \ddot{\underset{\sim}{\eta}} + m\psi_B^T C^T \tilde{\omega}^T \tilde{e} \underset{\sim}{\omega} + m\psi_B^T C^T \tilde{e} \dot{\underset{\sim}{\omega}} + \psi_B^T C^T \bar{\Phi} \ddot{\underset{\sim}{q}} + (m\psi_B^T + \bar{\psi}^T)\underset{\sim}{g} + K_C \underset{\sim}{\eta} = \psi_B^T C^T \underset{\sim}{F} \tag{33a}$$

$$I_0 \dot{\underset{\sim}{\omega}} + \tilde{\omega}^T I_0 \underset{\sim}{\omega} + \tilde{\Phi} \ddot{\underset{\sim}{q}} + \tilde{\omega}^T \tilde{\Phi} \dot{\underset{\sim}{q}} + J(\dot{\underset{\sim}{\omega}})\underset{\sim}{q} + J(\underset{\sim}{\omega})\dot{\underset{\sim}{q}} + \tilde{\omega}^T J(\underset{\sim}{\omega})\underset{\sim}{q}$$

$$+ \tilde{e}^T C\psi_B \ddot{\eta} m + \widetilde{[C g]}(e m + \bar{\Phi} q) = M \qquad (33b)$$

$$M\ddot{q} + \tilde{\Phi}^T \dot{\omega} + \bar{\Phi}^T C\psi_B \ddot{\eta} + \int_{m_A} \Phi^T \tilde{\omega}^T \tilde{a} \, dm_A \, \omega + \bar{\Phi}^T C g + K q = Q \qquad (33c)$$

Higher-order terms have been neglected and some nonlinear terms retained in Eq. (33b), consistent with the case of the spacecraft in space. In this case, the Euler angles vector α is assumed to be of arbitrary magnitude, which is responsible for many nonlinear terms in Eqs. (33).

4. Simulation and Control

The nonlinear equations of motion take the same basic form for the orbiting spacecraft as for the laboratory experiment, as can be seen above. Hence, the solution techniques suggested here apply for both situations.

Consider a first-order perturbation in the vectors R and ω.

$$R = R_0 + R_1, \; \omega = \omega_0 + \omega_1 \qquad (34)$$

where the first-order terms R_1 and ω_1 are small compared to the zero-order terms R_0, ω_0. Introducting Eqs. (34) into the nonlinear equations of motion, Eqs. (18) or (33), we obtain zero-order and first-order equations. The zero-order solutions R_0 and ω_0 can be obtained from an open-loop, rigid body maneuver strategy. These solutions can then be inserted into the first-order equations yielding linear equations with known time-varying coefficients. Simulation requires numerical integration. The equations can be put into their most compact form by approximating the first-order motion by means of a linear combination of eigenvectors corresponding to the lower frequencies. A control technique suppressing elastic vibration as well as rigid-body motion

that deviates from the desired maneuver, can also be formulated considering the equations in this compact form.

5. Conclusions

The equations of motion for the structure both in orbit about the earth and in the laboratory are nonlinear, even when the elastic deformations are small. The nonlinear terms result from the large rigid-body maneuver. For instance, centrifugal and Coriolis terms affecting the elastic displacements can be significant in a minimum-time maneuver, so that they must be retained in the equations. Through a perturbation approach, the nonlinear equations of motion can be transformed into a set of equations governing the rigid-body motions and a set of time-varying, linear equations governing small deviations from the prescribed rigid-body maneuver and elastic motion. Future work will include applying this perturbation approach to the above equations of motion for use in both simulation and control of the structure.

6. Reference

1. L. Meirovitch, Computational Methods in Structural Dynamics, Sijthoff-Noordhoff, The Netherlands, 1980.

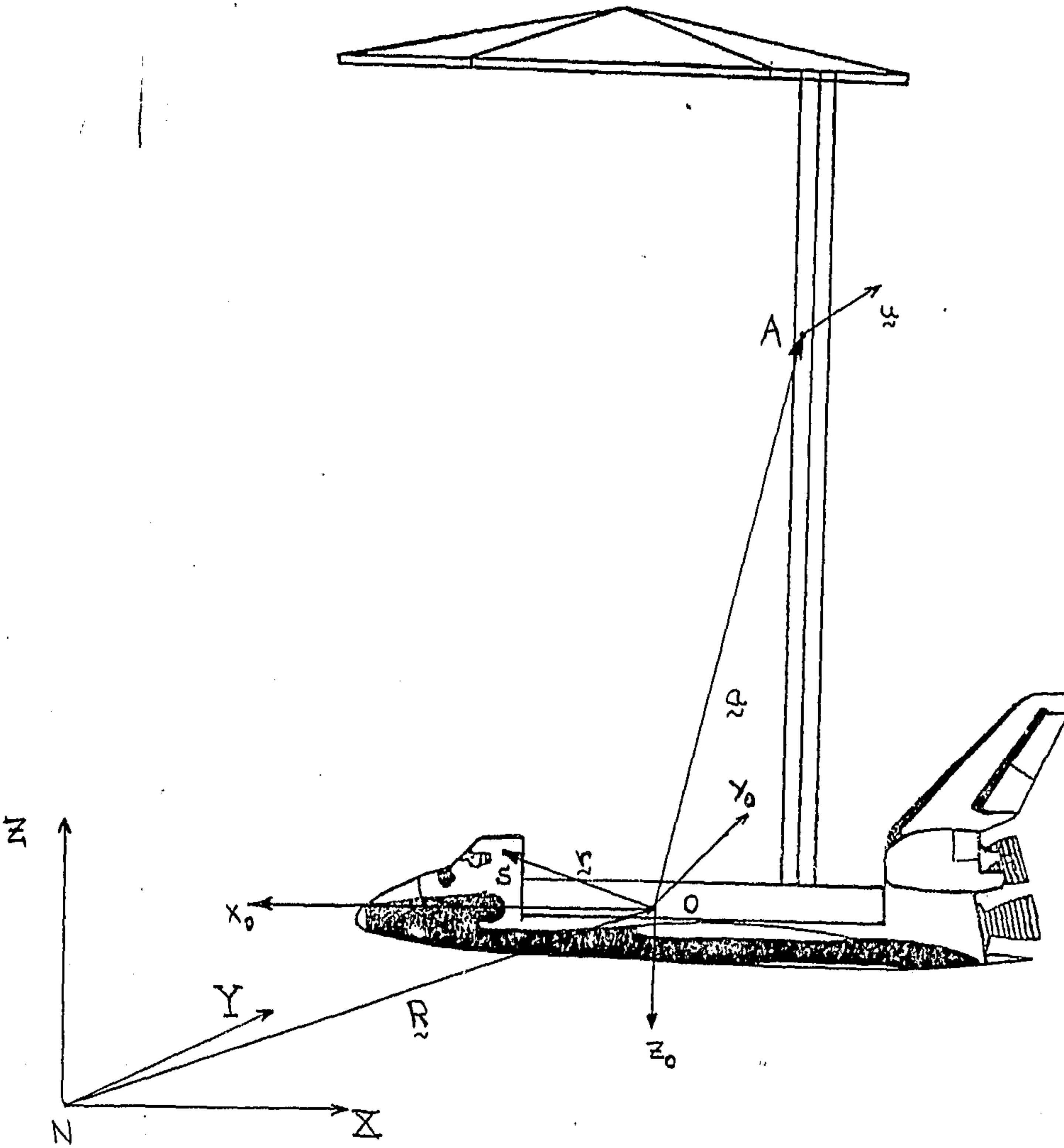

Figure 1 - SCOLE Configuration

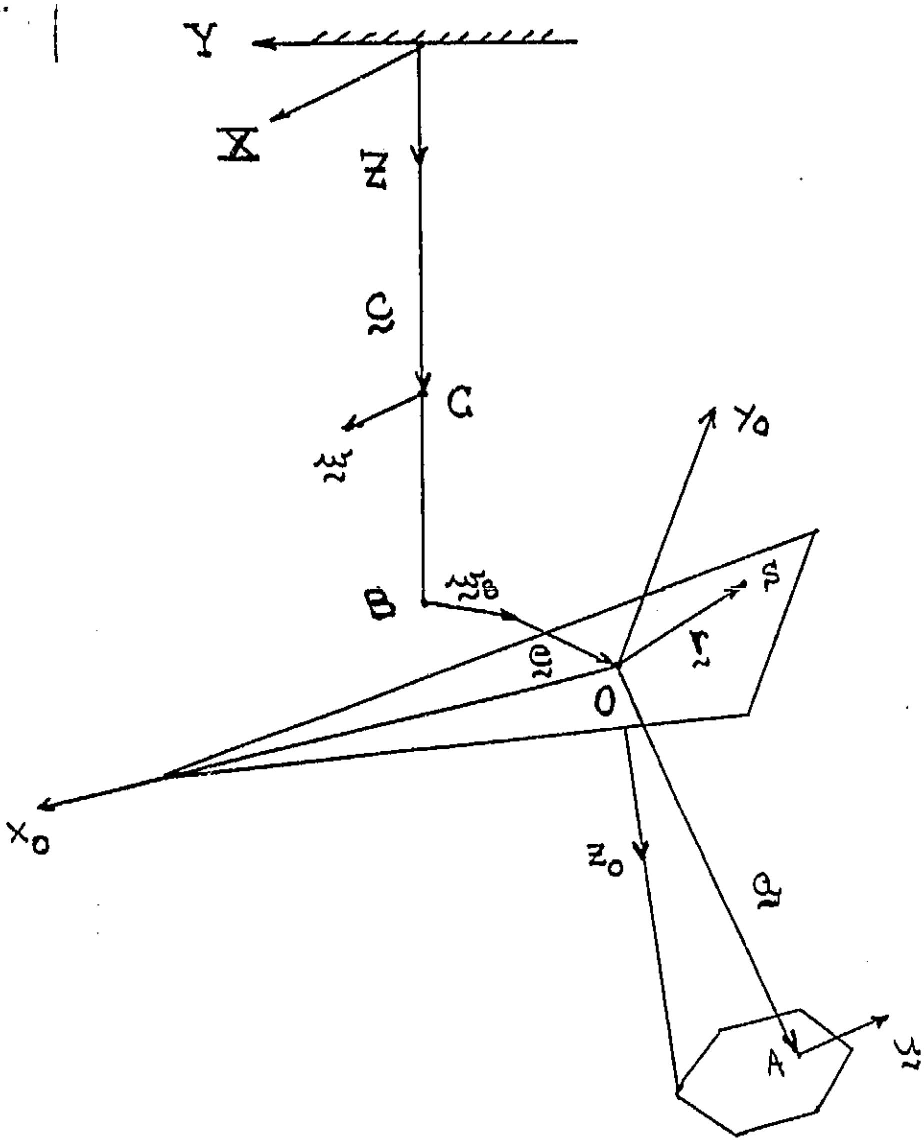

Figure 2 - Laboratory Model